Workshop
Affine Algebraic Geometry

DIJON - October 3-4 2011

Schedule

Monday 14h00-14h50, Salle René Baire :

Separating invariants for additive group actions
Émilie DUFRESNE (Universität Basel)

Monday 15h00-15h50, Salle René Baire :

L'algèbre des inverses unilatéraux d'une algèbre de polynômes (d'après V. Bavula)
Jean-Philippe FURTER (Université La Rochelle)

Monday 16h15-16h45, Salle René Baire :

Russell's hypersurface from a geometric point of view
Isac HEDÉN (Uppsala Universitet)

Tuesday 9h00-9h50, Salle René Baire :

Sur le groupe des transformations symplectiques du plan complexe
Jérémy BLANC (Universität Basel)

Tuesday 10h30-11h20, Salle René Baire :

Jacobiennes intermédiaires et transformations de $\mathbb{P}^3$
Stéphane LAMY (Université Toulouse III)

Tuesday 13h30-14h20, Salle René Baire :

On the Topologies on ind-Varieties
Immanuel STAMPFLI (Universität Basel)

Tuesday 14h30-15h20, Salle René Baire :

On automorphism ind-groups of affine schemes
Adrien DUBOULOZ (IMB Dijon)

Separating invariants for additive group actions
Émilie DUFRESNE (Universität Basel)
Abstract : Rather than considering the whole ring of invariants, one can consider a separating set, that is, a set of invariants whose elements separate any two points which can be separated by invariants. Part of the appeal of this new approach is that separating invariants can have better structural and computational properties than the ring of invariants. For example, there always exists a finite separating set, even when the ring of invariants is not finitely generated. In this talk, we discuss recent results concerning separating invariants for actions of the additive group in characteristic zero.

L'algèbre des inverses unilatéraux d'une algèbre de polynômes (d'après V. Bavula)
Jean-Philippe FURTER (Université La Rochelle)
Abstract : Soit K un corps de caractéristique 0. Par définition, la K-algèbre $\mathbb{S}_n$ est engendrée par les $2n$ éléments $x_1, \ldots, x_n, y_1, \ldots, y_n$ satisfaisant les relations

$$y_1 x_1 = 1, \ldots, y_n x_n = 1,\ [x_i, y_j] = [x_i, x_j] = [y_i, y_j] = 0,\ \forall\, i \neq j.$$

Nous étudions quelques propriétés de cette algèbre en les comparant notamment à celles de l'algèbre des polynômes $P_n = K[x_1, \ldots, x_n]$ et de l'algèbre de Weyl $A_n = K[x_1, \ldots, x_n, \partial_{x_1}, \ldots, \partial_{x_n}]$.

Russell's hypersurface from a geometric point of view
Isac HEDÉN (Uppsala Universitet)
Abstract : It was shown by Makar-Limanov himself that the Makar-Limanov invariant of Russell's hypersurface X is nontrivial and thus X is only diffeomorphic, but not isomorphic to affine 3-space. Russell's hypersurface is the affine variety in $\mathbb{C}^4$ defined by $x + x^2 y + z^3 + t^2 = 0$, and the Makar-Limanov invariant is by definition equal to the intersection of all kernels of locally nilpotent derivations on its coordinate ring. While Makar-Limanov's methods were mainly algebraic, the aim of this talk is to give a geometric argument from which we also obtain the result that $ML(X)$ is nontrivial. We consider X as an open part of a blowup $M \longrightarrow \mathbb{C}^3$, and show that every $\mathbb{C}^+$-action on X descends to $\mathbb{C}^3$. This in turn follows from the fact that any nontrivial $\mathbb{C}^+$-action on X induces a nontrivial $\mathbb{C}^+$-action on $W := Sp(B)$, where B is the graded algebra associated to the filtration of $\mathcal{O}(X)$ given by the pole order along the divisor $M \setminus X$ at infinity. The induced corresponding locally nilpotent derivation is homogenous, and we prove that the degree of any such derivation is negative – the result concerning the ML-invariant follows from this.

Sur le groupe des transformations symplectiques du plan complexe
Jérémy BLANC (Universität Basel)
Abstract : Une transformation symplectique du plan complexe $\mathbb{C}^2$ est une transformation birationnelle qui préserve la forme différentielle canonique $dx/x \wedge dy/y$. Je tâcherai d'expliquer pourquoi ce groupe est engendré par les applications monomiales et par une application spéciale d'ordre 5 $(x, y) \dashrightarrow -- > (y, (y+1)/x)$, résultat conjecturé par A. Usnich.

Jacobiennes intermédiaires et transformations de $\mathbb{P}^3$
Stéphane LAMY (Université Toulouse III)
Abstract : La question suivante m'a été posée par J. Sebag début 2010 : étant donnée f une transformation birationnelle de $\mathbb{P}^3$, est-il vrai que les surfaces contractées par f et f^{-1} sont les mêmes à équivalence birationnelle près ? J'essaierai de vous (me) convaincre que je sais répondre par la positive. L'argument utilise en particulier la notion de jacobienne intermédiaire.
Exercice préparatoire suggéré : répondre à la question similaire en dimension 2.

On the Topologies on ind-Varieties
Immanuel STAMPFLI (Universität Basel)
Abstract : In the 1960s Shafarevich introduced ind-varieties in order to explore certain naturally occurring groups that allow the structure of an infinite-dimensional analogon of an algebraic group (such as the group of polynomial automorphisms of $\mathbb{C}^n$). Shafarevich defined an ind-variety as the successive limit of an increasing chain

$$X_1 \subseteq X_2 \subseteq X_3 \subseteq \ldots$$

of varieties X_n, each one closed in the next. There are essentially two ways of endowing such an ind-variety with a topology. One topology is naturally induced by the increasing chain of varieties and is due to Shafarevich. The other is naturally induced by the regular functions on the ind-variety and is due to Kambayashi. These topologies differ on a rather large class of ind-varieties. The aim of this talk is to give an idea of the proof of this result.

On automorphism ind-groups of affine schemes
Adrien DUBOULOZ (IMB Dijon)
Abstract : (Joint-work in progress with J.-P. Furter) It is a well-established folklore that it is possible to consider the group of polynomial automorphisms of an affine space over a field as an infinite increasing union of nested algebraic closed sub-varieties. In other word, this group carries the structure of an algebraic ind-group. The usual construction based on the work of Shafarevich and Kambayashi depends on the existence of universal bounds on the degree of the inverse of a polynomial automorphism. In particular, it is not at all clear that the resulting structure is the "good" or the "natural" one.
In this talk, I will first review the appropriate theoretical framework to formulate problems about ind-varieties. Then I will explain how to proceed to construct a canonical structure of algebraic ind-group on the automorphism group of an arbitrary affine variety. If time permits, I will indicate the nature of the additional ingredients needed to generalize this construction in the relative case of a "nice" affine fibration over an arbitrary base scheme.