Workshop
Automorphisms of Affines Spaces

DIJON - March 29-30 2010

Schedule

Monday 10h00-11h00, Room 317 :

Gizatullin surfaces with a $\mathbb{Q}$-trivial canonical class.
Hubert FLENNER (Universität Bochum)

Abstract : By a remarkable result of Bandmann and Makar-Limanov a smooth Gizatullin surface V with trivial canonical class can be embedded into the affine 3-space with equation $xy = P(t)$ for some polynomial $P(t)$. More recently this was generalized to normal surfaces by D. Daigle, where again the same result holds. We give a report about the following result of Kai Ledwig, a PhD student of mine.
Theorem : Let V be a normal Gizatullin surface with a $\mathbb{Q}$-trivial canonical class. Then V is isomorphic to a locally closed subset of a weighted projective space P of dimension 4. More precisely, V is isomorphic to a complement $D\backslash V$, where in suitable weighted homogeneous coordinates (x, y, s, z) the closed subvariety V has equation $xy = P(s, z)$ and D is the open subset $z \neq 0$.
The main step in the proof is to show that a normal Gizatullin surface with a $\mathbb{Q}$-trivial canonical class always admits a hyperbolic $\mathbb{C}^*$-action. Moreover it turns out that the DPD-presentation of such a surface is of a special type as studied in a recent paper of Kaliman, Zaidenberg and myself. Applying one of the main results of that paper the theorem follows.

Monday 14h00-15h00, Room 317 :

A birational characterization of affine varieties with a trivial ML invariant.
Alvaro LIENDO (Institut Fourier, Grenoble)

Abstract : In this talk we show the following birational classification of normal affine varieties with trivial Makar-Limanov invariant (ML invariant for short). Let X be an affine variety over an algebraically closed field $\mathbf{k}$ of characteristic 0. If $\mathrm{ML}(X) = \mathbf{k}$ then $X \simeq_{\mathrm{bir}} Y \times \mathbb{P}^2$ for some variety Y. Conversely, in any birational class $Y \times \mathbb{P}^2$ there is an affine variety X with $\mathrm{ML}(X) = \mathbf{k}$. We also propose a generalization of the ML invariant and we conjecture that the triviality of this new invariant implies rationality. We prove this conjecture in dimension at most three.

Thursday 15h30-16h30, Room 317 :

Automorphisms of Koras-Russel threefolds.
Lucy MOSER-JAUSLIN (Institut de Mathématiques de Bourgogne, Dijon)

Abstract : Let X be an hypersurface of $\mathbb{C}^4$ defined by an equation of the form $x^d y + z^k + t^l + x = 0$, where $d \geq 2$, $k < l$, and k and l are coprime. These varieties have been studied by Koras and Russell. They are smooth, contractible but not algebraically isomorphic to $\mathbb{C}^3$. The first case ($d = 2$, $k = 2$ and $l = 3$) is known as the Russell cubic. In this talk, I will present a description of the automorphism group of the Russel cubic and discuss the generalizations which leads to a complete description of these groups for the other Koras-Russell threefolds.

Tuesday 9h00-10h00, Room 317 :

Infinite-transitivity for cones over flag varieties.

Karine KUYUMZHIYAN (Institut Fourier, Grenoble)

Abstract : (joint work with Ivan V. Arzhantsev and Mikhail Zaidenberg)
Suppose that X is a normal affine cone over a partial flag variety G/P. We show that the group of its special automorphisms $SAut(X)$ acts m-transitively on $X \setminus 0$ for every integer m.

Tuesay 10h30-11h30, Room 317 :

Automorphisms of toric varieties.

Mikhail Zaidenberg (IF, Grenoble). *Mikhail Zaidenberg* (Institut Fourier, Grenoble)

Abstract : (joint work with Ivan Arzhantsev and Karina Kuyumzhiyan)
We show that the special automorphism group of a non-degenerate toric variety X acts $m-$transitively on X for every m.

Tuesday 14h00-15h00, Room 317 :

Danielewski fiber product trick for contractible threefolds.

Adrien DUBOULOZ (*Institut de Mathématiques de Bourgogne, Dijon*)

Abstract : In Danielewski famous counter-example to the Cancellation Problem for surfaces, the isomorphism between the cylinders over the surfaces under consideration is a consequence of the fact that they both admit the structure of a $\mathbb{G}_a$-bundle over a same scheme, namely, an affine line with a double origin. Recently, in a joint work with Lucy Moser-Jauslin and Pierre-Marie Poloni, we found a Cancellation counter-example for contractible affine threefolds. In this talk, I will explain how to interpret this new counter-example in term of a similar fiber product phenomenon, coming from the fact that these threefolds contains strictly quasi-affine open subsets which are total spaces of $\mathbb{G}_a$-bundles over a same algebraic space.