WORKSHOP
AUTOMORPHISMS OF AFFINES VARIETIES

DIJON - June 22-23 2009

SCHEDULE

Monday 10h30-11h30, IMB Room 318 :

Algebraic density property of homogeneous spaces.
Shulim KALIMAN (Miami University)

Abstract : A smooth affine algebraic variety X has the algebraic density property (ADP) if the Lie algebra generated by completely integrable algebraic vector fields coincides with the space of all algebraic vector fields on X. It has the algebraic volume density property (AVDP) when the similar fact holds for algebraic vector fields of zero divergence. We establish ADP for a wide class of affine algebraic varieties that includes most of linear algebraic groups and homogeneous spaces, and AVDP for a class of objects that includes, in particular, all semi-simple Lie groups.

Monday 11h45-12h45, IMB Room 318 :

Epimorphisms of Polynomial Rings defined using Generic Matrices.
Gene FREUDENBUG (Western Michigan University)

Abstract : Basic algebraic properties of polynomials defined by 2×2 generic matrices are established. Specializing to the determinant one case, we obtain a family of polynomials $F_n \in \mathbb{Z}[x, y]$ whose properties give rise to a natural family of epimorphisms $\Phi : \mathbf{k}[x_1, x_2, y_1, y_2, z] \to \mathbf{k}[u, v]$ over any field $\mathbf{k}$ of characteristic 0. The polynomials $F_n(x, y) \in \mathbb{Z}[x, y]$ are bivariate analogues of the Chebyshev polynomials $T_n(x)$ in many key respects, where $F_n(x, x) = T_n(x)$. We likewise obtain epimorphisms $\phi : \mathbf{k}[x, y, z] \to \mathbf{k}[t]$ defined by $\phi = (T_i, T_j, T_k)$. In particular, we determine which triples (i, j, k) produce surjections, and establish certain algebraic equivalences among them. For the knot enthusiast, when the ground field is real, this procedure supplies a vast array of knots parametrized by polynomials; a table of knot types associated with low-degree embeddings is given.

Monday 15h15-16h15, IMB Room 318 :

Group actions on affines cones.
Mikhail ZAIDENBERG (Institut Fourier, Université Grenoble I)

Abstract In a paper by Hubert Flenner and the speaker, the following question was raised. Consider the affine cubic 3-fold X given in $\mathbb{A}^4$ by equation $x_1^3 + x_2^3 + x_2^3 + x_2^3 = 0$. Does the coordinate ring of X admit a nonzero locally nilpotent derivation? More generally, in a recent preprint by Yuri Prokhorov, Takashi Kishimoto and the speaker we addressed the following question : *Determine the affine cones over smooth projective varieties which admit an action of a connected algebraic group different from the standard $\mathbb{C}^*$-action by scalar matrices and its inverse action.* We show in particular that all affine cones over smooth del Pezzo surfaces of degree ≥ 4 possess such an action. The original question whether this is also true for the affine cones over projective cubic surfaces, and in particular for the Fermat cubic cone above, occurs to be out of reach for our methods. Nevertheless, we provide a general geometric criterion that could be helpful also in this case.

Monday 16h30-17h30, IMB Room 318 :

Additive group actions on smooth affine surfaces.
Adrien DUBOULOZ (Institut de Mathématiques de Bourgogne, Université de Bourgogne)

Abstract A basic result due to R. Rentschler asserts that every algebraic action of the additive group $\mathbb{G}_a$ on the affine plane $\mathbb{A}^2$ is a twisted translation of the form $(x, y) \mapsto (x, y + tp(x))$ in a suitable coordinate system. It turns out that a similar description holds more generally for algebraic $\mathbb{G}_a$-actions on smooth affine surfaces S : every such action can be described locally in the étale topology as a twisted translation. This implies in particular that a categorical quotient $\rho : S \to S/\mathbb{G}_a$ always exists as a locally trivial $\mathbb{A}^1$-bundle over an algebraic space. I will present some applications of this description to the Cancellation Problem and to the question of instability of the Makar-Limanov invariant.

Tuesday 9h15-10h15, IMB Room 318 :

Relations between the leading terms of a polynomial automorphism.
Stéphane VÉNÉREAU (Mathematisches Institut, Universität Basel)

Abstract : This is joint work with P. Bonnet. The relations between the leading terms of a polynomial automorphism form an ideal. We prove the existence of a non-trivial locally nilpotent derivation preserving this ideal on the one hand and, on the other hand, when this ideal is principal, compute an upper bound for some degree function evaluated on the generator of this ideal. As application, we determine all the principal ideals of relations for automorphisms in dimension three.

Tuesday 10h45-11h45, IMB Room 318 :

$\mathbb{G}_a$-bundles over the Winkelmann quotient.
Dilia RUEDA (New Mexico State University)

Abstract : The Winkelmann quotient is an open subset of a smooth affine fourfold over which $\mathbb{C}^5$ is a principal $\mathbb{G}_a$-bundle. Our objective is to investigate similar constructed bundles in connection with possible counterexamples to the cancellation problem.

Tuesday 13h30-14h30, IMB Room 318 :

Field of $\mathbb{G}_a$-invariants.
David FINSTON (New Mexico State University)

Abstract : For k a field and G_a the additive group of k, an action of G_a on $X = k^n$ is called rationally triangulable if $k(X)$ has a generating set $x_1, ..., x_n$ for which the associated G_a action stabilizes the fields $k(x_1, ..., x_i)$ $i = 1, ..., n$. If k is algebraically closed of characteristic 0, rational triangulability of a G_a action on X is equivalent with rationalilty (i.e. pure transcendence over k) of the field $k(X)^{G_a}$ of G_a invariants. It is unknown whether these conditions hold for arbitrary actions and all n . In general though, the field of G_a invariants is ruled, and rational for arbitrary actions on k^n with $n < 5$. In another direction, one can ask whether the field $k(U)$ is ruled whenever k^n has an open subset of the form $U \times k^r$ with U an affine variety. This holds for $r = 1, 2$ and is unknown otherwise.

Tuesday 14h45-15h45, IMB Room 318 :

Affine $\mathbb{T}$-varieties, locally nilpotent derivations and singularities.
Alvaro LIENDO (Institut Fourier, Université Grenoble I)

Abstract : Let $X = \operatorname{spec} A$ be a normal affine variety endowed with an effective action of a torus $\mathbb{T}$ of dimension n. Let also ∂ be a homogeneous locally nilpotent derivation on the normal affine $\mathbb{Z}^n$-graded domain A, so that ∂ generates a $\mathbb{C}_+$-action on X. In this talk we will show some classification result of pairs (X, ∂) with special emphasis in the case where the orbits $\mathbb{C}_+$-action are contained in the orbit closures of the $\mathbb{T}$-action. In particular, we exhibit a family of singular non-rational varieties with trivial Makar-Limanov invariant. We will also give some results concerning singularities of $\mathbb{T}$-varieties.