Workshop
Automorphisms of Affines Spaces

BASEL - November 12-13 2009

Schedule

Thursday 14h00-14h50, Stapfelberg 7 :

! UNUSUAL LOCATION ! : Meeting at the Math. Institute (Rheinsprung 21) at 13h50.

Degrees, Newton Polygons and Algebraic Dependence.
Stéphane VÉNÉREAU (Universität Basel)

Abstract : The Newton polygon of a polynomial $p = \sum p_{ij}x^iy^j$ is the convex hull of its support, $\{(i,j)|p_{ij} \neq 0\}$, in the plane $\mathbb{R}^2$. A very elementary proof shows that if two polynomials are algebraically dependent then their Newton polygons are (almost) homothetic. In this talk we will present this proof and wonder to what extent it can be generalized.

Thursday 15h10-16h00, Stapfelberg 7 :

Equivariant embeddings of smooth affine varieties.
Jonas BUDMIGER (Universität Basel)

Abstract : It is well known that a smooth affine n-dimensional variety X admits a closed immersion into affine $2n+1$-space $\mathbf{A}^{2n+1}$. If furthermore $\varphi\colon X \to \mathbf{A}^{2n+2}$ and $\psi\colon X \to \mathbf{A}^{2n+2}$ are two closed immersions, then they are even equivalent in the sense that there exists an algebraic automorphism Φ of the ambient affine space $\mathbf{A}^{2n+2}$ such that $\psi = \Phi \circ \varphi$. What happens if we try to translate these statements into the equivariant setup ? Let now G be a linearly reductive group acting morphically on a smooth affine variety X. We then study G- equivariant embeddings of X into G-modules V. First, one sees that every affine G-variety can be embedded equivariantly into a suitable finite-dimensional G-module V. However, the bad news is that one cannot give an upper bound for the equivariant embedding dimension as in the non-equivariant case. But the good news is : The equivalence result stated above can be carried over in a natural way to the equivariant setup.

Thursday 16h20-17h10, Kleiner Hörsaal :

Le groupe d'automorphismes d'une famille de variétés de Koras-Russell.
Lucy MOSER-JAUSLIN (Institut de Mathématiques de Bourgogne, Dijon)

Abstract : Soit X une hypersurface de $\mathbf{C}^4$ définie par l'équation $x^dy+z^k+t^l+x=0$, où $d \geq 2$, $k < l$, et k et l sont premiers entre eux. Ces variétés ont été étudié par Koras et Russell. Elles sont contractibles, lisses, mais pas algébriquement isomorphes à $\mathbf{C}^3$. Le premier cas, ($d=2$, $k=2$ et $l=3$) est appelé le cubique de Russell. Dans cet exposé, je vais décrire un travail en collaboration avec A. Dubouloz et P.M. Poloni sur les automorphismes du cubique de Russell, et en plus comment on peut généraliser les arguments pour obtenir une description des automorphismes des autres variétés de Koras-Russell.

Thursday 18h30 Maths. Institute : **DRINK**

Thursday 19h30 : **Diner**

Friday 9h10-10h, Stapfelberg 7 :

! UNUSUAL LOCATION ! : Meeting at the Math. Institute (Rheinsprung 21) at 9h00.

Automorphismes de surfaces réelles.
Jérémy BLANC (Universität Basel)

Abstract : Recently, Kollár and Mangolte proved that one can approximate any diffeomorphism of a rational compact real surface by a diffeomorphism which is birational. In consequence, the study of the diffeomorphisms of such surfaces is reduced to the study of birational diffeomorphisms, which is more simpler. We will provide natural generators and relations of the group of birational diffeomorphisms of the simpler rational compact real surfaces, i.e., the sphere, the torus and the projective plane. We will explain how the situation differs for the Klein bottle and other rational surfaces, with a little discussion on the affine plane.

Friday 10h30-12h00, Kleiner Hörsaal :

Normal subgroup generated by a plane polynomial automorphism.
Jean-Philippe FURTER (Laboratoire MIA, Université de La Rochelle)

Abstract : This is a joint work with Stéphane Lamy. We study the normal subgroup $\langle f \rangle_N$ e generated by an element $f \neq id$ in the group G of complex plane polynomial automorphisms having Jacobian determinant 1. On one hand if f has length at most 8 relatively to the classical amalgamated product structure of G, we prove that $\langle f \rangle_N = G$. On the other hand if f is a sufficiently generic element of even length at least 14, we prove that $\langle f \rangle_N \neq G$. The technics employed are from 'combinatorial group theory'. More precisely, we will use Bass-Serre theory dealing with trees and amalgams and Lyndon-Schupp theory associating diagrams in the Euclidean plane to some products in amalgamated products. The lecture will be quite elementary and the used tools will be explained.

Friday 14h30-15h20, Stapfelberg 7 :

! UNUSUAL LOCATION ! : Meeting at the Math. Institute (Rheinsprung 21) at 14h20.

Surfaces de Danielewski et Conjecture Jacobienne.
Adrien DUBOULOZ (*Institut de Mathématiques de Bourgogne, Dijon*)

Abstract : Une surface affine S contenant un ouvert U isomorphe à $\mathbb{A}^2$ et munie d'un morphisme $S \to \mathbb{A}^2$ dont la restriction à U est étale constituerait un contre-exemple à la Conjecture Jacobienne en dimension 2. Il est classique que tout contre-exemple potentiel est en fait de ce type. Une construction géométrique de D. Wright elle-même inspirée des travaux de R. Peretz sur les valeurs asymptotiques des endomorphismes polynômiaux plans permet de limiter l'étude aux cas de surfaces S munies d'une $\mathbb{A}^1$-fibration générique $S \to \mathbb{P}^1$ à fibres irréductibles. Une adaptation de cette construction permet également de se ramener plutôt à des surfaces munies d'une $\mathbb{A}^1$-fibration générique au-dessus de la droite affine à deux origines. Dans ce contexte, j'expliquerais comment les résultats partiels de non existence obtenus par R. Peretz et D. Wright peuvent s'interpréter comme des obstructions cohomologiques 'naives' de nature purement différentielle. On verra en particulier pourquoi ce type de méthode ne permet pas de décider de manière immédiate si la surface de Danielewski d'équation $x^2z = y^2 - 1$ dans $\mathbb{A}^3$ admet ou non un morphisme étale vers le plan affine.

Friday 15h30-16h00, Stapfelberg 7 :

Some aspects of $\mathbb{A}^1$-bundles.
Immanuel STAMPFLI (Universität Basel)

Abstract : An $\mathbb{A}^1$-bundle is a morphism of varieties, which is locally of the form $\mathbb{A}^1_U \to U$, where $\mathbb{A}^1_U$ denotes the one-dimensional affine space over U. We want to study a condition for a morphism of varieties over $\mathbb{C}$ to be an $\mathbb{A}^1$-bundle. Namely we want to explain the following theorem and give an idea of its proof : Let $\varphi : X \to Y$ be an affine, flat morphism of varieties over $\mathbb{C}$ and assume that Y is irreducible and normal. If the fibers over all closed points of Y are non-empty and integral, and the general fiber is isomorphic to $\mathbb{A}^1$ then $\varphi : X \to Y$ is an $\mathbb{A}^1$-bundle.