

WORKSHOP
DERIVED CATEGORIES IN ALGEBRAIC GEOMETRY
 DIJON - May 23-25 2012

SCHEDULE

All lectures will take place in "Paris" lecture room at the ground floor of Mirande building.

Wednesday	Thursday	Friday
	MANIVEL/ABUAF I 10:00-11:00	MANIVEL/ABUAF III 10:00-11:00
	MACRI II 11:15-12:15	FAENZI 11:15-12:15
	Lunch Break 12:30-14:00	
MACRI I 14:00-15:00	STELLARI II 14:00-15:00	ADDINGTON 14:00-15:00
STELLARI I 15:15-16:15	BOLOGNESI 15:15-16:15	
Coffee Break		
BERNARDARA 16:45-17:45	MANIVEL/ABUAF II 16:45-17:45	
	19:30 Dinner	

Mini-courses

■ **Geometry and derived categories of cubic 3-folds and 4-folds**
 by *E. MACRI and P. STELLARI*

Abstract : The aim of these lectures is to recall some basic properties and classical results concerning the geometry of smooth cubic hypersurfaces of dimension 3 and 4. We explain a way to translate them in the language of derived categories of coherent sheaves and outline how this approach helps in solving geometric questions.

■ **Introduction to Homological Projective Duality (after Kuznetsov)**
 by *L. MANIVEL and R. ABUAF*

Abstract : These lectures will be an introduction to a circle of ideas due to A. Kuznetsov, aiming at understanding classical projective duality at the level of derived categories. We will discuss several examples, showing how homological projective duality can be a powerful tool for describing the derived categories of certain Fano or Calabi-Yau varieties, obtained as linear sections of homogeneous spaces.

Research Talks

- **Comparing Hassett's and Kuznetsov's Special Cubic Fourfolds**

Nicolas ADDINGTON

Abstract : Hassett described a countable set of Noether-Lefschetz divisors in the moduli space of cubic fourfolds, expecting a cubic to be rational if and only if it lies in one of these divisors. Kuznetsov described a K3 category associated to a cubic and conjectured the cubic is rational if and only if this category is the derived category of an honest, geometric K3 surface. In joint work with Richard Thomas, we can show that if the K3 category is geometric then the cubic lies in one of Hassett's divisors, and for cubics in a Zariski open subset of each divisor, the K3 category is geometric. (We are still working on showing that the subset is closed.) In particular we can show that for a very general cubic, the K3 category has no point-like objects.

- **Determinantal varieties and HPD**

Marcello BERNARDARA

Abstract : A. Bondal proposed the following problem: do the derived category of any smooth projective variety embed fully faithfully into the derived category of a Fano variety? In this talk I will present a joint work with M. Bolognesi and D. Faenzi, describing the homological projective duality between Segre and (categorical resolutions of) determinantal varieties. As a consequence, we get a list of varieties with a positive answer to Bondal's question. As an application, the categorical resolution of singularities of determinantal cubic 3-folds and 4-folds can be explicitly described.

- **Rational cubic fourfolds containing a plane with nontrivial Clifford invariant**

Michelle BOLOGNESI

Abstract : In this talk I will showcase a general class of smooth rational cubic fourfolds X containing a plane whose associated quadric surface bundle does not have a rational section. Equivalently, the Brauer class \mathcal{B} of the even Clifford algebra over the discriminant cover (a K3 surface S of degree 2) associated to the quadric bundle, is nontrivial. These fourfolds provide nontrivial examples verifying Kuznetsov's conjecture on the rationality of cubic fourfolds containing a plane. Indeed, using homological projective duality for grassmannians, one obtains another K3 surface S' of degree 14 and a nontrivial twisted derived equivalence $A_X = D^b(S; B) = D^b(S')$, where A_X is Kuznetsov's residual category associated to the cubic hypersurface X .

- **On the derived category of the Cayley plane**

Daniele FAENZI

Abstract : Many rational homogeneous varieties admit full strongly exceptional collection (Beilinson, Kapranov, Orlov, Kuznetsov, Fonarev), and even Lefschetz collections consisting of homogeneous bundles. Although this conjecturally holds for all such varieties, still many cases are missing. I will show a construction of such a collection for the Cayley plane, the minimal orbit of the exceptional group E_6 . The method involves restriction to quadrics in the Cayley plane, combined with a rephrasing of an idea of Bondal-Orlov. If time allows, I will present some hints on how to construct Lefschetz collections on some other homogeneous varieties. This work is in collaboration with Laurent Manivel.