Arithmetics of non-positively curved varieties

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Karim Adiprasito: Some birational news (using tools from geometric group theory)

The Oda conjecture states that every two birational toric varieties have a common iterated blowup. I will present the recent proof of the rational/weighted version of this conjecture, providing a constructive algorithm that reaches a common blowup using tricks I learned long ago to understand CAT(0) spaces. If time permits, I will also discuss relations to the solution of the semistable reduction conjecture. Joint work with Igor Pak.

Michela Artebani: Cox rings of surfaces with nef anticanonical class.

Let X be a smooth complex projective rational surface with $q(X) = 0$ and nef anticanonical divisor $-K_X$. The Cox ring [1]

$$R(X) = \bigoplus_{[D]\in \mathrm{Cl}(X)} \Gamma(X, \mathcal{O}_X(D))$$

encodes the effective divisor theory of X. The ring is known to be finitely generated when $-K_X$ is big [4,6] or when $\kappa(-K_X) = 1$ and the effective cone is polyhedral [3]. In this talk we present a uniform and explicit description of generators for $R(X)$ that depends only on the anticanonical geometry of X (the linear system $|-K_X|$ and the configuration of negative curves). Our construction extends the known descriptions for generalized del Pezzo surfaces [4,5] and for extremal rational elliptic surfaces [2]. This is joint work with Sofía Pérez Garbayo [7].

References:

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[2] M. Artebani, A. Garbagnati, A. Laface, Cox rings of extremal rational elliptic surfaces, Trans. Amer. Math. Soc. 368 (2016), no. 3, 1735–1757.
[3] M. Artebani and A. Laface, Cox rings of surfaces and the anticanonical Iitaka dimension, Adv. Math. 226 (2011), no. 6, 5252–5267, DOI 10.1016/j.aim.2011.01.007.
[4] V.V. Batyrev and O.N. Popov, The Cox ring of a del Pezzo surface, Arithmetic of higher-dimensional algebraic varieties (Palo Alto, CA, 2002), 2004, pp. 85–103.
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[7] M. Artebani, S. Pérez Garbayo, Cox rings of nef anticanonical rational surfaces, Adv. Math. 475 (2025).

Claudio Bravo: Homotopy invariance of homology groups of certain arithmetic groups

The classical homotopy invariance theorem in algebraic K-theory states that $K_i(R) \cong K_i(R[t])$, for any regular ring R. This motivated the study of unstable analogues for the homology of linear algebraic groups. In this setting, the homotopy invariance question asks whether, for a given algebraic group G and a ring R, the canonical map $G(R) \to G(R[t])$ induces isomorphisms:

$$H_*(G(R), M) \;\cong\; H_*(G(R[t]), M).$$

Important contributions were made by Soulé, Knudson and Wendt, who analyzed actions of arithmetic groups on Bruhat–Tits buildings to establish positive results in several cases, mainly for isotrivial groups.

In this talk, we will first review these classical results. Then, we will concentrate on recent progress concerning quasi-split but non-isotrivial group schemes, focusing in particular on the case of the special unitary group SU_3 over $\mathbb{P}^1$. As time permits, we will also discuss conjectural results for arbitrary group schemes defined over $\mathbb{P}^1$ and possible approaches to proving this conjecture for quasi-split groups of arbitrary rank.

Ana-Maria Castravet: Gale duality, blow-ups and moduli spaces

I will discuss joint work with Carolina Araujo, Inder Kaur and Diletta Martinelli about the birational geometry of blow-ups of projective spaces at points in general position. We will explore Gale duality, a correspondence between sets of $n = r + s + 2$ points in projective spaces $\mathbb{P}^r$ and $\mathbb{P}^s$. For small values of s, this duality has a remarkable geometric manifestation: the blow-up of $\mathbb{P}^r$ at n points can be realized as a moduli space of vector bundles on the blow-up of $\mathbb{P}^s$ at the Gale dual points.

Fabián Levicán: Embeddings of weighted projective spaces

Let X be a projective variety of dimension n and let $\mathcal{L}$ be an ample line bundle on X. For $k \geq 0$, it is generally difficult to determine if $\mathcal{L}^{\otimes k}$ is very ample and if it additionally gives a projectively normal embedding. In the case of projective toric varieties, both properties are respectively equivalent to the *very ampleness* and *normality* of the corresponding polytopes, and by a result of Ewald-Wessels are classically known to hold for $k \geq n - 1$.

We study embeddings of weighted projective spaces $\mathbb{P}(a_1, \ldots, a_n)$ via their corresponding rectangular simplices $\Delta(a'_1, \ldots, a'_n)$. Weighted projective spaces provide some of the simplest examples of singular toric Fano varieties, which makes them a natural testing ground for such problems. We give multiple criteria (depending on arithmetic properties of the weights a_i) to obtain bounds for the power k which are sharp in many cases. We also introduce combinatorial tools that allow us to systematically construct families exhibiting extremal behaviour. These results extend earlier work of Payne, Hering and Bruns-Gubeladze.

This is joint work in progress with Praise Adeyemo and Dominic Bunnett.

Enrica Floris: On algebraic subgroups of the Cremona group

The study of connected algebraic subgroups of the Cremona group is a classical way of deepening the understanding of the Cremona group. Via the Weil regularisation theorem and the Minimal Model Program, to such a group we associate a rational Mori fibre space on which it acts regularly. In this talk, we will discuss the notion of maximal connected algebraic subgroups of the Cremona group, and its relation with the geometry of the associated Mori fibre spaces. This is a work in collaboration with A. Fanelli and S. Zimmermann.

Alvaro Liendo: Nash Blowup Fails to Resolve Singularities in Dimensions Four and Higher

Resolution of singularities in characteristic zero was established by Hironaka, whose approach relies on a sequence of blowups along carefully chosen centers. Although effective, this method involves a variety of non-canonical choices. In order to obtain a canonical procedure, Nash proposed the blowup that now bears his name, raising the question of whether its iteration suffices to resolve all singularities.

In joint work with Federico Castillo, Daniel Duarte, and Maximiliano Leyton-Álvarez, we construct counterexamples showing that in dimensions at least four the iterated Nash blowup does not lead to a resolution. I will discuss these counterexamples and place them in the broader context of the resolution problem.

Pedro Montero: Counting rational points on Hirzebruch-Kleinschmidt varieties

This talk is about asymptotic formulas for the number of rational points of bounded (or large) height on Hirzebruch-Kleinschmidt varieties over global fields. These varieties are realizations of split toric varieties with Picard rank 2, and their explicit models enable computations that go beyond the general expectation of the Manin-Peyre conjecture. This is joint work with Tobías Martínez (Universidad de El Salvador) and Sebastián Herrero (Universidad de Santiago de Chile).

Lucas Moulin: Real forms of complexity-one varieties

Given a complex algebraic variety X, a classical problem in algebraic geometry is to determine the real forms of X. We are interested in a slightly different case : The case of G-varieties, where G is a complex algebraic group, and their real forms which are equivariant under the action of a real algebraic group F which is a real form of G. The complexity of a G-variety is the codimension of a B-orbit in general position, with B any Borel subgroup of G. The varieties of complexity 0 are called spherical varieties, which include the well-known toric varieties. Over algebraically closed fields, complexity-one varieties have a combinatorial classification due to Timashev. We will illustrate how this can be extended to arbitrary perfect fields by considering real forms of SL_2-threefolds containing a dense G-orbit and of completions of SL_3/T.

Ana Julisa Palomino: Automorphisms of prime power order of weighted hypersurfaces

Let $X = V(F)$ be a hypersurface of $\mathbb{P}^{n+1}_a$ given as the zero set of a homogeneous form $F \in \mathbf{C}[x_0, ..., x_{n+1}]$ of degree d. The group of linear automorphisms, denoted by $\mathrm{Lin}(X)$, is the subgroup of $\mathrm{Aut}(X)$ of automorphisms that extends to an automorphism of the ambient space $\mathbb{P}^{n+1}_a$ i.e.,

$$\mathrm{Lin}(X) = \{\phi \in \mathrm{Aut}(\mathbb{P}^{n+1}_a) : \phi(X) = X\}.$$

Recently [Ess24], shows that under certain conditions, we have that group $\mathrm{Lin}(X) = \mathrm{Aut}(X)$, and furthermore, $\mathrm{Aut}(X)$ is finite. In this talk, we will apply this result to compute all the possible primes numbers that appear as the order of an automorphism of a well-formed quasi-smooth hyperspace in a weighted projective space. This is a generalization of [GAL13].

References
[Ess24] Louis Esser, Automorphisms of weighted projective hypersurfaces, arXiv: 2301.07872
[GAL13] González-Aguilera, Víctor and Liendo, Alvaro, On the order of an automorphism of a smooth hypersurface, arXiv: 1102.3443

Yulieth Prieto: Projective Hyperkähler Manifolds with Large Picard Number

We study the intersections of Hassett divisors in the moduli space of smooth special cubic fourfolds, with the aim of constructing examples of hyperkähler manifolds deforming to Hilbert schemes of points on K3 surfaces. Our focus is on cases with Picard number at least four, where we show that the resulting hyperkähler manifolds are always isomorphic to moduli spaces of twisted sheaves on K3 surfaces. In contrast, we point out that when the Picard number is three, one can obtain a variety of lines associated with a cubic fourfold that is never birational to any moduli space of twisted sheaves on a K3 surface.

Andriy Regeta: Solvable subgroups in the automorphism group

I will discuss the following result, based on a joint work with Cantat, Kraft and van Santen: Let G be a solvable group of automorphisms of an affine variety. If G is generated by an irreducible family of automorphisms containing the identity, then G is an algebraic group. As an application, I present the following statement: a closed connected solvable subgroup of the group of automorphisms $\mathrm{Aut}(X)$ of an algebraic variety X has derived length less than or equal to $\dim X + 1$ and the equality holds iff X is isomorphic to the affine space $\mathbb{A}^n$. Moreover, all Borel subgroups (maximal connected solvable subgroups) of $\mathrm{Aut}(\mathbb{A}^n)$ of derived length $n+1$ are conjugate and there are many non-conjugate Borel subgroups of $\mathrm{Aut}(\mathbb{A}^n)$.