THE 24TH AFFINE ALGEBRAIC GEOMETRY MEETING (HIROSAKI)

ABSTRACTS OF TALKS

⋆ 4th March (Wednesday)

■ Hirokazu Nasu (Tokai University, Hiratsuka):

Title: **Deformations of space curves lying on a complete intersection $K3$ surface of genus 5**

Abstract: The Hilbert schemes of rational curves on Fano varieties have been widely studied, because rational curves on Fano varieties tend to deform well and play an important role in the classification of Fano varieties. In contrast, much less is known about the Hilbert schemes of curves of higher genus. This is mainly because deformation obstructions are harder to control, and the geometry of the Hilbert scheme becomes more complicated. In particular, when obstructions do not vanish, it is often difficult to determine even the local dimension of the Hilbert scheme. In this talk, we review some results on deformations of curves lying on a $K3$ surface contained in a smooth Fano threefold. As a higher-dimensional analogue, we study deformations of space curves in $\mathbb{P}^5$ lying on a $K3$ surface $S_{2,2,2} \subset \mathbb{P}^5$, which is a smooth complete intersection of three quadrics.

■ Takuzo Okada (Kyushu University, Fukuoka):

Title: **Birational geometry of Fano 3-fold weighted complete intersections: I**

Abstract: I will explain known results on Fano 3-fold weighted complete intersections with a special emphasis on the rationality problem and the birational rigidity type problem. In the first talk (on 5th March), I will explain relevant notions such as birational rigidity/solidity and give an overview of known results on weighted hypersurfaces. In the second talk, I will explain recent updates on on-going studies of birational geometry of Fano 3-fold weighted complete intersections of codimension 2.

■ In-Kyun Kim (KIAS, Seoul, Korea):

Title: **Cylindricity of varieties in weighted projective spaces**

Abstract: Cylindricity plays an important role in the affine and birational geometry of Fano varieties. Cylindrical normal projective varieties have been intensively studied over the last decade, motivated by their relationship with polar cylinders and with additive group actions on generalized affine cones. In this talk, I first review cylindricity for weighted projective spaces, emphasizing accessible criteria and illustrative examples. I then turn to weighted hypersurfaces and weighted complete intersections, presenting explicit examples that highlight how cylindricity interacts with the geometry of the ambient weighted projective space.

■ Guillaume Kineider (Aix-Marseille Université, Marseille, France):

Title: **Topology of some real Fano threefolds**

Abstract: Given a smooth real projective variety, its real locus, endowed with the Euclidean topology, is a smooth closed manifold. Certain topological and geometric properties of this manifold reflect subtle algebro-geometric features of the underlying variety. For instance, Comessatti showed in 1914 that, in dimension two, having an orientable hyperbolic real locus obstructs $\mathbb{R}$-rationality. In a series of papers, Kollár used the Minimal Model Program machinery to classify the geometric structures carried by the real loci of most threefolds with Kodaira dimension $-\infty$, leaving open only the case of the finitely many families of Fano threefolds. For most of the latter, the question remains unsolved. In this talk, I will discuss the topological classification of the smooth real Fano varieties obtained as blow-ups of $\mathbb{P}^3$ along certain real rational curves.

■ Masayoshi Miyanishi (Kwansei Gakuin University, Sanda):

Title: **Normal subalgebras of algeraic and analytic rings: Subrings of** $\mathbb{C}[x, y, z]$

Abstract: Let A be a finitely generated subalgebra of a polynomial ring $k[x, y, z]$ over the field k of complex numbers. Assuming that A is normal, we clarify the structure of A under additional assumptions if $\dim A \geq 2$. If $\dim A = 2$ and A is regular, then Spec$A$ has an $\mathbb{A}^1$-fibration over $\mathbb{P}^1$ or $\mathbb{P}^1$ with restrictions on the multiplicities of singular fibers. If $\dim A = 3$, we assume that A is cofinite, i.e., $k[x, y, z]$ is a finite A-module, and A contains a coordinate x. Then either A is a polynomial ring or the invariant subring $k[x, u, v]^G$ with respect to a small finite subgroup G of $\mathrm{GL}(2, k)$. This result was published in J. Pure and Applied Algebra, which is available onlone, 2024.

⋆ 5th March (Thursday)

■ Masayoshi Miyanishi (Kwansei Gakuin University, Sanda):

Title: **Normal subalgebras of algeraic and analytic rings: Subrings of** $\mathbb{C}\{x, y, z\}$

Abstract: The results for normal subalgebras of a polynomial ring $\mathbb{C}[x, y, z]$ in the first talk (on 4th March) are extended to those in a convergent power series ring $\mathbb{C}\{x, y, z\}$, and local complex analogues are proved for complex spaces). A topological characterization of $\mathbb{C}^2/G$ with a finite group G due to Gurjar-Shastri is given by a new conceptual proof. This result will be published in Tohoku Mathematical Journal.

■ Takuzo Okada (Kyushu University, Fukuoka):

Title: **Birational geometry of Fano 3-fold weighted complete intersections: II**

Abstract: *See the abstract of Takuzo Okada on 4th March.*

■ Dae-Won Lee (Ewha Womans University, Seoul, Korea):

Title: **Rationality and cylindricity of** $\Bbbk$**-forms of singular del Pezzo surfaces**

Abstract: The study of cylinders in normal projective varieties is of significant interest due to their intrinsic link to unipotent group actions on affine algebraic varieties. Over a field $\Bbbk$ of characteristic zero, it is known that cylindricity in lower-dimensional varieties (appearing as generic fibers) implies the existence of vertical cylinders in higher-dimensional fibered spaces.

In this talk, for a field $\Bbbk$ of characteristic zero, we give a criterion for the rationality and cylindricity of $\Bbbk$-forms of a singular del Pezzo surface obtained as a blow-up of a weighted projective plane. This is a joint work with In-Kyun Kim and Masatomo Sawahara.

■ Kento Fujita (The University of Osaka, Toyonaka):

Title: **Another view on smooth prime Fano threefolds of degree 22 with infinite automorphism groups: I**

Abstract: All smooth Fano threefolds with infinite automorphism groups are understood due to Prokhorov, Kuznetsov and Shramov by use of deep studies of their Hilbert scheme of lines. I will present as our joint work with Adrien Dubouloz and Takashi Kishimoto an alternative and self-contained proof of it, allowing us to use several properties on the smooth quintic del Pezzo threefold. Moreover, I would like to explain an interesting elementary link joining prime Fano threefolds of degree 22 with Fano threefolds of No. 2.21 in Mori-Mukai's list.

■ In-Kyun Kim (KIAS, Seoul, Korea):

Title: **Oscillation index of oscillatory integrals and the real log canonical threshold of a real analytic function**

Abstract: This talk, based on joint work with Morihiko Saito, investigates the relationship between oscillatory integrals and singularity invariants of real analytic functions. For a real analytic function f, the oscillation index describes the leading exponents in the asymptotic expansion of associated oscillatory integrals, while the real log canonical threshold $\mathrm{rlct}(f)$ measures the singularity of f via resolution of singularities. These invariants are closely related: it is known that the oscillation index is always bounded above by $-\mathrm{rlct}(f)$, and it has long been expected that equality holds in natural settings, such as the Newton nondegenerate case. In this talk, I will present new results showing that this expectation fails in general. More precisely, for convenient homogeneous Newton-nondegenerate polynomials of even degree in even dimension, the inequality is strict.

⋆ 6th March (Friday)

■ Hiromichi Takagi (Gakushuin University, Tokyo):

Title: **Sarkisov link for prime $\mathbb{Q}$-Fano 3-folds related with Type II projections**

Abstract: Reid and Papadakis developed several unprojection techniques that produce $\mathbb{Q}$-Fano 3-folds of higher codimension from ones of lower codimension. If one turns this viewpoint around and looks instead at projections, it leads to a way of constructing Sarkisov links for $\mathbb{Q}$-Fano 3-folds. In this talk, I will explain how to build Sarkisov links related to so-called Type II projections for $\mathbb{Q}$-Fano threefolds of codimension 3 and 4. Following Brown-Zucconi, we use toric Sarkisov links, although some extra ideas are needed in our situation. I would also like to stress that constructions via key varieties are very helpful for describing these Sarkisov links.

■ Kento Fujita (The University of Osaka, Toyonaka):

Title: **Another view on smooth prime Fano threefolds of degree 22 with infinite automorphism groups: II**

Abstract: *See the abstract of Kento Fujita on 5th March.*

■ Teppei Takamatsu (Saitama University, Saitama):

Title: **Fano threefolds of genus** 12 **with large automorphism group in positive and mixed characteristic**

Abstract: We study prime Fano threefolds of genus 12 with positive-dimensional automorphism groups in positive and mixed characteristic. In characteristic 0, such varieties have been classified into three types: Mukai-Umemura type, $\mathbb{G}_a$-type, and $\mathbb{G}_m$-type. We prove that the same classification holds in positive characteristic. Moreover, we exhibit several phenomena specific to positive characteristic; for example, a Mukai-Umemura threefold over a field k exists if and only if char$k$ is not 2 or 5. Finally, we explain an application to the Shafarevich conjecture, concerning finiteness of varieties over rings of integers.

■ Jaehyun Kim (Ewha Womans University, Seoul, Korea):

Title: **Relative cones of Du Val del Pezzo surfaces of degree one**

Abstract: The minimal resolutions of Du Val del Pezzo surfaces admit various $\mathbb{P}^1$-fibrations depending on the singularity type. These fibrations yield natural $\mathbb{A}^1$-bundle structures on the surfaces with an ample boundary. In this talk, we focus on Du Val del Pezzo surfaces of degree 1 with Picard rank 2 that arise as compactifications of an $\mathbb{A}^1$-bundle over an affine curve. This talk is based on joint work with Dae-Won Lee and Masatomo Sawahara.