THE 23RD AFFINE ALGEBRAIC GEOMETRY MEETING (NIIGATA)

ABSTRACTS OF TALKS

⋆ 4th March (Tuesday)

■ Takuzo Okada (Kyushu University, Fukuoka):

Title: **Birational geometry of sextic double solids**

Abstract: A double cover of the projective 3-space branched along a sextic surface is called a sextic double solid. It is a classical result of Iskovskikh that a smooth sextic double solid X is birationally superrigid, that is, it cannot be birational to a Mori fiber space other than X itself and the birational automorphism group coincides with the automorphism group. Cheltsov and Park generalized this result to sextic double solids with nodes. In my talk, after explaining these backgrounds, I will talk about further generalizations to sextic double solids admitting terminal singularities of type cA. This talk is based on a joint work with Igor Krylov and Erik Paemurru.

■ Masatomo Sawahara (Hirosaki University, Hirosaki):

Title: **Minimal compactifications of the affine plane with only star-sharped singularities**

Abstract: Letting X be a normal compact complex surface and let Γ be a closed curve on X, we say that the pair (X, Γ) is a minimal compactification of $\mathbb{C}^2$ if $X \backslash \Gamma \simeq \mathbb{C}^2$ and Γ is irreducible. Kojima and Takahashi classified minimal compactifications of $\mathbb{C}^2$ with at most lc singularities. As a corollary, every minimal compactification of $\mathbb{C}^2$ with at most lc singularities has only star-shaped singularities. In this talk, we classify minimal compactifications of $\mathbb{C}^2$ with only star-shaped singularities. As an application, we construct many examples of minimal compactifications of $\mathbb{C}^2$ with a non-lc singularity, whose anti-canonical divisor is numerically ample.

■ Masayoshi Miyanishi (Kwansei Gakuin University, Sanda):

Title: **Descending chain condition for finite morphisms of algebraic varieties (with R.V. Gurjar)**

Abstract: The descending chain condition ((DCC), for short) for finite surjective morphisms of algebraic varieties belonging to a category $\mathcal{C}$ asserts that for any descending chain with X_i and f_i being objects and morphisms in $\mathcal{C}$,

$$X_1 \xrightarrow{f_1} X_2 \longrightarrow \cdots\cdots \to X_n \xrightarrow{f_n} X_{n+1} \longrightarrow \cdots$$

there exists an integer $N > 0$ such that f_n is an isomorphism for every $n \geq N$. We discuss if the (DCC) holds for various subcategory $\mathcal{C}$ of the category of algebraic varieties, especially, the category of algebraic surfaces.

■ Niklas Lemcke (Waseda University, Tokyo):

Title: **Vanishing of nef and big Witt-divisorial sheaves in positive characteristic**

Abstract: We generalize previous results about duality of Witt-divisorial sheaves. As an application we show vanishing in the nef and big case in dimension two.

■ Masaru Nagaoka (Gakushuin University, Tokyo):

Title: **Completion of the affine 3-space into sextic del Pezzo fibrations**

Abstract: Yu. G. Prokhorov constructed examples of completions of the affine 3-fold $\mathbb{A}^3$ into smooth 3-folds endowed with sextic del Pezzo fibrations in his study on singular Fano 3-folds of genus 12. By construction, the boundary divisor consists of a fiber and a horizontal divisor which is non-normal along a section. In this talk, we explain how to construct new examples of completions of $\mathbb{A}^3$ into smooth 3-folds endowed with sextic del Pezzo fibrations, whose horizontal divisors are normal along any sections.

⋆ 5th March (Wednesday)

■ Hirokazu Nasu (Tokai University, Hiratsuka):

Title: **Stably degenerate and obstructed curves on a del Pezzo surface**

Abstract: J. O. Kleppe gave a conjecture concerning a maximal family of curves lying on a smooth cubic surface in $\mathbb{P}^3$, which generalizes Mumford's example of a generically non-reduced irreducible component of the Hilbert scheme. A version of the conjecture (modified by Ph. Ellia) essentially says every linearly normal space curve (numerically) contained in that surface is stably degenerate, i.e., every small global deformation of such a curve is still contained in a (deformation of) a cubic surface. We formulate a similar conjecture for space curves contained in a smooth del Pezzo surface of degree greater than 3. We discuss the degree 4 case of the conjecture, i.e., curves contained in a smooth complete intersection of two hyperquadrics in $\mathbb{P}^4$.

■ Joonyeong Won (Ewha Womans University, Seoul, Korea):

Title: **On singular del Pezzo surfaces embedded in weighted projective spaces**

Abstract: The smooth del Pezzo surfaces are among the most familiar, and fundamental, objects in algebraic geometry. As a generalization of it, we discuss some properties of singular del Pezzo surfaces embedded in weighted projective spaces, in particular, K-stability and the existence of K-polar cylinder.

■ Ayako Kubota (Saitama University, Saitama):

Title: **Some examples of the invariant Hilbert scheme of the nilpotent orbit closures of type** A

Abstract: The invariant Hilbert scheme parametrizes affine schemes with an action of a reductive algebraic group. If the parameter is chosen correctly, one obtains a projective morphism, called the quotient-scheme map, from the invariant Hilbert scheme to an affine quotient variety. The quotient-scheme map is an isomorphism over an open subset, so the

invariant Hilbert scheme is a candidate for a resolution of singularities of the affine quotient variety. In this talk, we look at some examples of the invariant Hilbert scheme associated to nilpotent orbit closures of type A.

■ Ryuji Tanimoto (Shizuoka University, Shizuoka):

Title: **Homomorphisms from $\mathrm{SL}(2,k)$ to $\mathrm{SL}(4,k)$ in positive characteristic**

Abstract: As a continuation of our study of exponential matrices, we are interested in $\mathrm{SL}(2,k)$-actions on complete algebraic varieties in positive characteristic. And then, we begin our study on homomorphisms from $\mathrm{SL}(2,k)$ to $\mathrm{SL}(n,k)$ in positive characteristic. In this talk, we show homomorphisms from $\mathrm{SL}(2,k)$ to $\mathrm{SL}(4,k)$ in positive characteristic. We might discover new representations of $\mathrm{SL}(2,k)$ in positive characteristic.

■ Kayo Masuda (Kwansei Gakuin University, Sanda):

Title: **A smooth acyclic affine fourfold with a free $\mathbb{G}_a$-action and an equivariant $\mathbb{A}^3$-fibration**

Abstract: Let X be a factorial complex affine variety of dimension ≥ 3 with an algebraic action of the additive group $\mathbb{G}_a$. Let $\pi : X \to Y$ be the algebraic quotient morphism where we assume Y is an affine variety. When π is faithfully flat, we investigate π by $\mathbb{G}_a$-equivariant affine modifications and give criteria for π to be a trivial $\mathbb{A}^1$-bundle. As a consequence, for a smooth acyclic fourfold X with a free $\mathbb{G}_a$-action and a $\mathbb{G}_a$-equivariant $\mathbb{A}^3$-fibration $f : X \to \mathbb{A}^1$ where $\mathbb{G}_a$ acts trivially on $\mathbb{A}^1$, we give a criterion for the algebraic quotient Y to be isomorphic to $\mathbb{A}^3$ with f as a coordinate and $X \cong Y \times \mathbb{A}^1$.

⋆ 6th March (Thursday)

■ Paul Alexander Helminck (Tohoku University, Sendai):

Title: **Non-linear complex matroids, hyperplane arrangements and pair-of-pants decompositions**

Abstract: For a set of complex hyperplanes in n-dimensional projective space, we can associate a combinatorial object known as the homogeneous complex matroid of the arrangement. It has n+1 different dehomogenizations, and each of these describes a good cover of the hyperplane complement, so that we can recover the topology of the complement from standard nerve theorems. These covers are however not compatible for different dehomogenizations, so that they are ill-suited for gluing.

In this talk, we describe a universal good cover of a hyperplane complement that is independent of the chosen dehomogenization. The idea is to consider angles of quotients of hyperplane equations. The induced combinatorial structure lands outside the realm of ordinary complex and oriented matroids, as the different relations are non-linear. We thus call this object a non-linear complex matroid.

I will give the proof that this defines a good cover, as well as connections to wonderful compactifications and Kato-Nakayama spaces. If time permits, I will show how one can glue these objects to obtain generalized pair-of-pants decompositions of smooth complex varieties.

■ Daisuke Matsushita (Hokkaido University, Sapporo):

Title: **On deformation of pairs of symplectic varieties and Lagrangian subvarieties**

Abstract: Since the late 2010s, there has been increasing interest in considering higher-dimensional generalizations of K3 surfaces by extracting two key properties: the existence of a holomorphic symplectic form and the fact that the Albanese map is always trivial. In real symplectic geometry, it is common to study Lagrangian submanifolds, and they are also an important research subject in algebraic geometry.

In this talk, we will discuss what can be said when viewing the pair of a symplectic manifold and a Lagrangian submanifold from the perspective of deformations, as well as the possible applications of this viewpoint.

■ Kenta Hashizume (Niigata University, Niigata):

Title: **On minimal model program for log canonical pairs in complex analytic setting**

Abstract: Remarkable progress has been made in recent years in the field of the minimal model theory for complex algebraic varieties. The first breakthrough was brought by Birkar, Cascini, Hacon and McKernan. In 2022, Fujino generalized their results to projective morphisms between complex analytic spaces. This is the first step of the minimal model theory in the complex analytic setting. In this talk, I will introduce recent progress of the minimal model theory for log canonical pairs in complex analytic setting. This talk contains joint works with Makoto Enokizono.

■ Takanori Nagamine (Nihon University, Tokyo):

Title: **On an algebra whose quotient field is retract rational**

Abstract: Let K/k be a field extension. K is said to be retract rational over k if there exists a k-domain A such that its quotient field is K and it is a retract of a localized polynomial ring $B := k[x_1, \ldots, x_n, f^{-1}]$, that is, there are k-algebra homomorphisms $B \to A$ and $A \to B$ such that their composition is identity on A. It is well known that if k is an infinite field, then every rational field is retract rational. However, the converse does not hold in general.

In this talk, we study conditions for retract rational fields to be rational. For this purpose, we focus on a k-algebra A which is a retract of $k[x_1, \ldots, x_n, f^{-1}]$. In this case, the quotient field of A is retract rational. When f is a monomial, we provide a classification of such algebras under certain conditions. As a corollary, we show that every retract of a Laurent polynomial ring is again a Laurent polynomial ring. Furthermore, we show that Zariski's cancellation problem for algebraic tori holds affirmatively for any dimension and any field.

This talk is based on joint work with Neena Gupta (arXiv:2301.12681).