

**THE 22ND AFFINE ALGEBRAIC GEOMETRY MEETING  
(NIIGATA)**

ABSTRACTS OF TALKS

★ 1st March (Wednesday)

■ Hideo Kojima (Niigata University, Niigata):

**Title: Log multicanonical systems of smooth affine surfaces of log Kodaira dimension one**

Abstract: In this talk, I give some results on log multicanonical systems of smooth affine surfaces of log Kodaira dimension one. Let  $S$  be a smooth affine surface of logarithmic Kodaira dimension one and let  $(V, D)$  be a pair of a smooth projective surface  $V$  and a simple normal crossing divisor  $D$  on  $V$  such that  $V \setminus \text{Supp } D = S$ . The main result of this talk is that, for any  $m \geq 8$ ,  $|m(K_V + D)|$  gives a ruled fibration  $\Phi$  such that  $F \cdot D = 2$ , where  $F$  is a fiber of  $\Phi$ .

■ Makiko Mase (Tokyo Metropolitan University, Tokyo):

**Title: On lattices and  $K3$  surfaces admitting symplectic automorphism action**

Abstract: Let  $X$  be a  $K3$  surface admitting a (finite) symplectic automorphism group  $G$ . The quotient  $X/G$  has at most ADE singularities and its minimal model is again a  $K3$  surface. Consider the lattice  $L$  generated by all the classes of  $(-2)$ -curves in the exceptional divisors of the singularity in  $X/G$ . Following Nikulin's study, we discuss whether or not there exists a primitive closure of  $L$ . This is a progressing joint work with Dr. Kenji Hashimoto.

■ Masaru Nagaoka (Gakushuin University, Tokyo):

**Title: Completion of the affine 3-space into del Pezzo fibrations**

Abstract: Although  $\mathbb{P}^2$  is the unique completion of the affine plane  $\mathbb{A}^2$  into a smooth projective surface of Picard rank one, there are several completions of the affine 3-space  $\mathbb{A}^3$  into smooth projective 3-folds of Picard rank one, which were classified by Furushima, Nakayama, Peternell, Schneider, et.al. When the completions are allowed to have singularities and larger Picard ranks, the number of completions is expected to become larger. In this talk, we consider the case where the completions are  $\mathbb{Q}$ -factorial terminal 3-folds endowed with del Pezzo fibrations. I will explain how to construct such compactifications by way of pencils of divisors to show that every smooth del Pezzo surface can appear as a closed fiber of such del Pezzo fibrations. This is a joint work with A. Dubouloz and T. Kishimoto.

■ Takanori Nagamine (National Institute of Technology, Oyama College, Oyama):

**Title: Factorial varieties with a torus action of complexity one**

Abstract: An affine algebraic variety whose coordinate ring is a unique factorization domain often appears in various problems of affine algebraic geometries. For example, the Zariski cancellation problem, the finiteness problem for a  $\mathbb{G}_a$ -invariant ring, etc. In this talk, we consider a factorial variety of dimension  $d$  with a  $(d-1)$ -dimensional torus action. Mori (1977) and Ishida (1977) classified them for dimensions  $d = 2, 3$ , respectively. Hausen, Herppich and Süß (2011) classified them for all dimensions in the case where the characteristic of the ground field is zero. Our main result shows that this class is identical to the class of affine varieties defined by trinomials, thus generalizing their earlier work.

This is a joint work with Gene Freudenburg (Western Michigan University).

■ Buddhadev Harja (Tata Institute of Fundamental Research, Bombay, India):

Title: **Eilenberg-MacLane Spaces in Algebraic Surface Theory**

Abstract: In algebraic topology, an Eilenberg-MacLane space is a path-connected topological space with possibly a single non-trivial homotopy group. Classifying all smooth complex algebraic Eilenberg-MacLane varieties is an interesting yet seemingly challenging question. Recently, in collaboration with R. V. Gurjar and S. R. Gurjar, we prove a classification theorem for smooth non-contractible complex affine Eilenberg-MacLane surfaces of (log) non-general type. We also prove a result about the universal cover of smooth complex projective Eilenberg-MacLane surfaces assuming an affirmative answer to an old conjecture of Igor R. Shafarevich. I will display these results in my talk.

A classification theorem for smooth complex projective Eilenberg-MacLane surfaces of non-general type has been recently found by S. R. Gurjar and P. Pokale which I will mention in my talk for the sake of completeness about the above classification question for algebraic surfaces. I would touch upon other related results if time permits.

★ 2nd March (Thursday)

■ Akinari Hoshi (Niigata University, Niigata):

Title: **Rationality problem of two-dimensional quasi-monomial group actions**

Abstract: The rationality problem of two-dimensional purely quasi-monomial actions was solved completely by Hoshi, Kang and Kitayama (2014). As a generalization, we solve the rationality problem of two-dimensional quasi-monomial actions under the condition that the actions are defined within the base field. In order to prove the theorem, we give a brief review of the Severi-Brauer variety with some examples and rationality results. We also use a rationality criterion for conic bundles of  $\mathbb{P}^1$  over non-closed fields. This is a joint work with Hidetaka Kitayama (arXiv:2012.12046).

■ Benjamin Collas (RIMS, Kyoto University, Kyoto):

Title: **Homotopic Galois theory: arithmetic principles**

Abstract: Homotopic Galois theory deals with the arithmetic and geometric symmetries of spaces, such as embodied in a unique avatar that is Grothendieck's étale fundamental group (a birational invariant that yet does not fully characterize equivalence classes).

In their classical flavors, homotopic Galois theory and affine algebraic geometry can be seen as intersecting via Hilbert irreducibility theorem and Noether's problem for the regular inverse Galois problem (Dèbes, Fried, Harbater et al. '90s) and birational anabelian geometry (Pop

1995). This talk will focus on some arithmetic principles (e.g., the rigidification of geometry and invariants) and some various anabelian reconstructions techniques which are in relation with birational geometry and group actions and at the heart of recent arithmetic developments – e.g., special loci (C. 2010-), absolute reconstruction, GT group, new indeterminacies and beyond ring geometry (Hoshi and Mochizuki 2010-).

By raising the question “how does arithmetic open geometric frontier?”, this talk is a first attempt at building bridges between “Arithmetic and Homotopic Galois Theory” and “Affine Algebraic Geometry”.

■ Masatomo Sawahara (Hirosaki University, Horosaki):

Title: **Some results on cylinders in Du Val del Pezzo surfaces**

Abstract: A cylinder in an algebraic variety means its open subset, which is isomorphic to the direct product of the affine line and an algebraic variety. In this talk, we will present the following two results:

(A) Let  $S$  be a Du Val del Pezzo surface of degree  $\geq 2$  defined over an algebraically closed field of characteristic 0. If  $S$  contains an  $(-K_S)$ -polar cylinder, then  $S$  contains  $H$ -polar cylinder for every ample  $\mathbb{Q}$ -divisor  $H$  on  $S$ .

(B) Let  $S$  be a Du Val del Pezzo surface defined over an algebraically closed field of characteristic 2. If the Dynkin type of  $S$  is  $7A_1$ ,  $8A_1$  or  $D_4 + 4A_1$ , then  $S$  does not contain any cylinder.

■ Jaehyun Kim (Ewha Womans University, Seoul, Korea):

Title: **A Conjecture on  $K$ -stability and Cylindricity**

Abstract: There is a conjecture by I. Cheltsov, J. Park, Y. Prokhorov and M. Zaidenberg that regards the relationship between the  $K$ -stability and cylindricity on Fano varieties. They expected that Fano varieties with at worst klt singularities without any anticanonical polar cylinder are  $K$ -polystable. In this talk, we answer to the conjecture by considering certain singular del Pezzo hypersurfaces in three dimensional weighted projective spaces. This is joint work with In-Kyun Kim and Joonyeong Won.

■ Masayoshi Miyanishi (Kwansei Gakuin University, Sanda):

Title: **Geometrically pure  $\mathbb{G}_a$ -action**

Abstract: We propose the definition of an action  $\sigma$  of the additive group scheme  $\mathbb{G}_a$  on an affine variety  $Y$  to be geometrically pure, which ensures the existence of a geometric quotient of  $Y$  by the  $\mathbb{G}_a$ -action  $\sigma$  if  $Y$  is normal. Namely there exists the quotient morphism  $q : Y \rightarrow X$  to a normal affine variety  $X$  such that the graph morphism  $\Psi : \mathbb{G}_a \times Y \rightarrow Y \times_X Y$  is an isomorphism. Geometric pureness of the given  $\mathbb{G}_a$ -action is the first criterion ever to guarantee the existence of a geometric quotient  $Y/\mathbb{G}_a$ . As a consequence, we obtain an algebraic characterization of the affine 3-space.

★ 3rd March (Friday)

■ Kayo Masuda (Kwansei Gakuin University, Sanda):

**Title: Singular loci of  $\mathbb{A}^1$ -fibrations induced by  $\mathbb{G}_a$ -actions on affine 3-space**

Abstract: Let  $X$  be an affine variety defined over  $\mathbb{C}$  with a nontrivial algebraic  $\mathbb{G}_a$ -action and let  $\pi : X \rightarrow Y$  be the algebraic quotient morphism by the  $\mathbb{G}_a$ -action. Then the general closed fiber of  $\pi$  is a  $\mathbb{G}_a$ -orbit  $\mathbb{A}^1$  and  $\pi$  is an  $\mathbb{A}^1$ -fibration. When  $X$  is an affine 3-space, then  $\pi$  is surjective and  $Y$  is an affine plane. We observe the singular locus of  $\pi$  consisting of points  $y \in Y$  such that  $\pi^*(y)$  is not isomorphic to  $\mathbb{A}_{k(y)}^1$ .

■ Yuri Yatagawa (Tokyo Institute of Technology, Tokyo):

**Title: Partially logarithmic characteristic cycles and index formula**

Abstract: We consider a computation for the characteristic cycle of a constructible étale sheaf on a smooth variety, which is defined by Beilinson-Saito with vanishing cycles, in terms of ramification theory. For this purpose, we introduce an algebraic cycle called "partially logarithmic characteristic cycle" for a rank one sheaf using ramification theory as a candidate for a computation and we would like to discuss the index formula for the candidate.

■ Pedro Montero (Universidad Técnica Federico Santa María, Valparaiso, Chile):

**Title: On strictly elliptic  $K3$  surfaces and del Pezzo surfaces**

Abstract: It is a well-established fact that complex Enriques surfaces can be represented as quotients of  $K3$  surfaces through an involution group action without fixed points. From this classical geometrical construction, which arises from considering cyclic coverings, we delve into the study of  $K3$  surfaces as double covers of rational surfaces. In this talk, we will consider involutions on  $K3$  surfaces whose fixed locus consists of a single smooth curve of general type. We will show that it is possible to study, following Kodaira's classification of singular fibers, non-trivial elliptic fibrations on such  $K3$  surfaces induced by conic bundles on del Pezzo surfaces. These conic bundle structures on del Pezzo surfaces have traditionally been studied using techniques from Mori theory. Consequently, these results enable us to obtain new insights into the classification of elliptic fibrations on  $K3$  surfaces admitting non-symplectic involutions. This is a joint work with Paola Comparin (Temuco, Chile), Yulieth Prieto (Trieste, Italy) and Sergio Troncoso (Turin, Italy).