

THE 21ST AFFINE ALGEBRAIC GEOMETRY MEETING

ABSTRACTS OF TALKS

★ 3rd March (Friday)

■ Masaru Nagaoka (Gakushuin University, Tokyo):

Title: **Log del Pezzo surfaces without tigers in low characteristics**

Abstract: Log del Pezzo surfaces are 2-dimensional Fano varieties with klt singularities, which form a building block in the minimal model program. Keel-McKernan and Lacini classified log del Pezzo surfaces of Picard number one in characteristic different from two and three. Their strategy is based on whether or not there are special divisors called tigers over such surfaces. In this talk, I will explain the classification result on log del Pezzo surfaces of Picard number one without tigers in characteristic two and three.

■ Sho Tanimoto (Nagoya University, Nagoya):

Title: **Non-free sections of Fano fibrations**

Abstract: Manin's Conjecture predicts the asymptotic formula for the counting function of rational points over number fields or global function fields. In the late 80's, Batyrev developed a heuristic argument for Manin's Conjecture over global function fields, and the assumptions underlying Batyrev's heuristics are refined and formulated as Geometric Manin's Conjecture. Geometric Manin's Conjecture is a set of conjectures regarding properties of the space of sections of Fano fibrations, and it consists of three conjectures: (i) Pathological components are controlled by Fujita invariants; (ii) For each nef algebraic class, a non-pathological component which should be counted in Manin's Conjecture is unique (This component is called as Manin component); (iii) Manin components exhibit homological or motivic stability. In this talk we discuss our proofs of GMC (i) over complex numbers using theory of foliations and the minimal model program. Using this result, we prove that these pathological components are coming from a bounded family of accumulating maps. This is joint work with Brian Lehmann and Eric Riedl.

■ Hirokazu Nasu (Tokai University, Hiratsuka):

Title: **Primary obstructions to deforming embedded curves and an application to the Hilbert scheme of space curves**

Abstract: Computing obstructions is useful for determining the dimension and the singularity of a Hilbert scheme at a given point, but it is a hard task if the obstruction space is nonzero. Generalizing a technique developed in a joint research with S.Mukai, I have recently obtained a new sufficient condition for a first order deformation of a curve on a projective threefold to be primarily obstructed. In 1980s, J.O. Kleppe gave a conjecture concerning the dimension of the Hilbert scheme of space curves (lying on a smooth cubic surface). In this talk, as an application, I will talk about a recent progress on the conjecture.

■ Masatomo Sawahara (Saitama University, Saitama):

Title: Cylindrical ample cones of Du Val del Pezzo surfaces

Abstract: Polarized cylinders in normal projective varieties receive a lot of attention from the viewpoint of connecting unipotent group actions on affine algebraic varieties. Hence, we shall focus on the configuration of cylindrical ample cones of normal projective varieties. Cheltsov, Park and Won studied cylindrical ample cones of smooth del Pezzo surfaces. In this talk, we will discuss the configuration of cylindrical ample cones of Du Val del Pezzo surfaces. As a result, if S is a Du Val del Pezzo surface of degree ≥ 3 such that $\text{Sing}(S) \neq \emptyset$, then S contains an H -polar cylinder for every ample \mathbb{Q} -divisor H on S .

★ 4th March (Saturday)

■ Ryuji Tanimoto (Shizuoka University, Shizuoka):

Title: Triangular $\mathbb{Z}/3\mathbb{Z}$ -actions on the affine four-space in characteristic three

Abstract: Let k be a field of positive characteristic p . A finite group action on the affine n -space \mathbb{A}_k^n over k is said to be *wild* if p divides the order of the finite group. Few examples of wild finite group actions on \mathbb{A}_k^n are known. We know a linear action of a p -cyclic group $\mathbb{Z}/p\mathbb{Z}$ on \mathbb{A}_k^n . Except for this linear example, we hardly know examples of wild $\mathbb{Z}/p\mathbb{Z}$ -actions on \mathbb{A}_k^n . So, we started to study constructing examples of wild $\mathbb{Z}/p\mathbb{Z}$ -actions on \mathbb{A}_k^n . In this talk, we will describe triangular $\mathbb{Z}/3\mathbb{Z}$ -actions on \mathbb{A}_k^4 in characteristic three, up to conjugation of automorphisms of \mathbb{A}_k^4 .

■ Karol Palka (Polish Academy of Sciences, Warsaw):

Title: What is almost minimal ?

Abstract: A minimal model of a quasi-projective surface and of a log surface with nonzero boundary can be singular. To avoid singularities, for log smooth surfaces with reduced boundary Miyanishi developed the notion of an almost minimal model. It is related to a minimal model by a well described morphism - a peeling - contracting only some curves supported in the boundary. We show that the idea of almost minimalization can be used more widely. Given a generalized log canonical or a \mathbb{Q} -factorial log surface (X, D) defined over an algebraically closed field of arbitrary characteristic we define its almost minimal model, whose underlying surface has singularities not worse than X and which differs from a minimal model by a peeling. We discuss properties of almost minimalization for boundaries of type rD , where D is reduced and r is a positive rational number.

■ Takuzo Okada (Saga University):

Title: Birational rigidity of certain singular Fano 3-folds

Abstract: It can happen that a special member of a family of Fano 3-folds fails to be birationally (super)rigid while its general members are birationally (super)rigid. A typical situation is when general members are smooth and some special member possesses a singular point from which one can construct a birational map (Sarkisov link) to another Mori fiber space. The aim of this talk is to explain this phenomena, and also explain birational rigidity of certain Fano 3-folds with only compound singularities of type A. This talk is based on a joint work with Krylov, Paemurru and Park.

■ Kenta Hashizume (Kyoto University, Kyoto):

Title: On effective base point freeness for klt pairs

Abstract: For a projective klt pair whose log canonical divisor is nef, the abundance conjecture predicts the semi-ampleness of the log canonical divisor. Assuming the abundance conjecture, then it is natural to study the integer m by which the multiple of log canonical divisor is base point free. This is called the effectivity of the base point freeness. In this talk, I will explain a recent result on this topic.

★ 5th March (Sunday)

■ Kayo Masuda (Kwansei Gakuin University, Sanda):

Title: Factorial affine \mathbb{G}_a -varieties with principal plinth ideals

Abstract: We study a factorial affine variety with an algebraic action of the additive group \mathbb{G}_a when its plinth ideal is principal. We analyze the quotient morphism by equivariant affine modifications.

■ Ayako Kubota (Waseda University, Tokyo):

Title: Hilbert scheme of the Cox realization of the nilpotent cone of type A

Abstract: The invariant Hilbert scheme is a moduli space of schemes which are stable under an action of a reductive algebraic group. By a suitable choice of the parameter, it becomes a candidate for a resolution of singularities of a quotient singularity. In the first half of this talk, I will explain two main problems in the study of the invariant Hilbert scheme from the point of view of birational geometry of singularities. One of them is about the invariant Hilbert scheme of the Cox realization of a singularity, and we discuss the case where the singularity is the nilpotent cone of type A in the second half of the talk.

■ Masayoshi Miyanishi (Kwansei Gakuin University, Sanda):

Title: Improvements of some of my past results

Abstract: I had chances to look back my past results and find them still having rooms for improvement. My interest lies in Abhyankar's problem which asks:

Let k be an algebraically closed field of characteristic $p > 0$. Let C be an affine plane curve defined by a polynomial $f(x, y) \in k[x, y]$. Is the curve C_α defined by $f = \alpha$ isomorphic to \mathbb{A}^1 for every $\alpha \in k$?

In the case of characteristic zero, this is the case. Furthermore, f is a ring generator of $k[x, y]$. But in the case $p > 0$, the last assertion does not hold by an example of Nagata. The problem remains unsolved more than fifty years. I will speak on the case C is liftable to the characteristic zero. More precisely, we have

Theorem. *Let (R, \mathfrak{m}) be a DVR. Let $k = R/\mathfrak{m}$ and $K = Q(R)$. Let $f \in R[x, y]$ be an element such that \bar{C} and C_K define respectively the curves isomorphic to \mathbb{A}^1 in \mathbb{A}_k^2 and \mathbb{A}_K^2 , where \bar{C} and C_K are defined by equating to the zero the following elements $\bar{f} := f \pmod{\mathfrak{m}} \in k[x, y]$ and $f_K := f \otimes 1 \in K[x, y]$. Then f is a ring generator over R . Namely, there exists $g \in R[x, y]$ such that $R[x, y] = R[f, g]$.*

This result implies that Abhyankar's Problem is solved positively if the given f is liftable to a ring generator in characteristic zero.

■ Andriy Regeta (University of Jena, Jena):

Title: On the automorphism group of an affine variety

Abstract: I will show that the automorphism group of an affine variety is essentially never isomorphic to a linear algebraic group as an abstract group. Further, I will show that the commutative connected subgroup of the automorphism group $\text{Aut}(X)$ of an affine variety X is the union of algebraic subgroups of $\text{Aut}(X)$. Using this as a tool I will show that an affine toric variety is determined by its automorphism group in the category of connected affine algebraic varieties over algebraically closed uncountable fields of any characteristic.